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NONSTATIONARY FLOW IN THE MODEL CHANNEL OF A RAMJET ENGINE IN PULSE-PERIODIC ENERGY SUPPLY

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A study has been made of the influence of the pulse-periodic supply of energy that is equal to the energy released in the combustion of hydrogen in air on the structure of supersonic flow in a channel of variable cross section, modeling the duct of a ramjet engine. The flow has been modeled on the basis of two-dimensional nonstationary gas-dynamic equations. Different flow regimes have been obtained depending on the configuration of the zones of energy supply and the excess-air coefficients.

Introduction. Pulse-periodic energy supply to a gas flow can be interpreted as the supply of radiant energy. It gives us hope for of extending the range of flight Mach numbers (above $M_\infty = 6$) for which we can use the ramjet channel as part of a compound engine (e.g., as part of a detonation MHD generator [1]) for increasing the effective specific impulse. It is expected that radiant-energy supply will be used in laser engines [2].

Below, we consider nonstationary flow of an ideal gas in a plane channel of variable cross section, modeling the element of a ramjet engine, in distributed energy supply. The energy supply is allowed for by the source term in the energy equation. Chemical reactions, change in the composition of the mixture, and the dependence of the heat capacity and other properties of the medium on temperature and pressure are disregarded. In many cases such a simplified formulation makes it possible to reveal the most substantial gasdynamic effects [3].

The zones of energy supply represent rectangles and (sweptback) corners.

The idea of investigating the influence of the shape of an energy-supply zone in the form of a corner stretched streamwise is related to the results [4] on study of the energy supply in a supersonic flow in zones strongly stretched streamwise. In this case we observe a continuous supersonic-to-subsonic transition without the formation of a normal shock ahead of the energy-supply zone (suspended shocks are present). In this connection, we have investigated, in the present work, the possibility of the supersonic flow existing in the channel shock waves or with a value of the Mach number that is at least average over the cross section and exceeding unity. The power of the supplied energy was assumed to be equal to the power of combustion of hydrogen with an excess-air coefficient of 1–2. The investigation carried out is a logical continuation of the investigations of [5, 6]. The results obtained were presented at the IX All-Russia Conference on Theoretical and Applied Mechanics [7]. Papers [8, 9] with analogous flow regimes in continuous combustion of hydrogen were presented at this conference.

Formulation of the Problem. We model nonstationary flow in a plane channel of a variable cross section with distributed energy supply. We solve the Euler equations in conservative form for a gas with a constant adiabatic exponent:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{Q} ,$$

$$\mathbf{U} = (\rho, \rho u, \rho v, e) , \quad \mathbf{F} = (\rho u, p + \rho u^2, \rho uv, u(p + e)) , \quad \mathbf{G} = (\rho v, \rho uv, p + \rho v^2, v(p + e)) ,$$

$$\mathbf{Q} = (0, 0, 0, q) .$$

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For the gas model in question, we have

$$p = (\gamma - 1)\,(e - 0.5\rho\,(u^2 + v^2))\,, \quad a^2 = \frac{\gamma p}{\rho}\,.$$

In pulse periodic energy supply, the quantity q is determined as

$$q = \Delta e\,(x, y)\,g\,(t)\,, \quad g\,(t) = \sum_m \delta\,(t - m\Delta t)\,.$$

These equations are solved in the channel of variable cross section. The channel shape and dimensions are indicated in the figures given below. The short initial portion of the channel ($0 \le x < 1$) has a constant cross section equal to unity. Then the channel smoothly diverges and becomes a chamber ($6 < x < 16$) in which energy supply is carried out ("combustion chamber;" its half-width is equal to two). After the chamber, the channel diverges and has outlet half-width equal to 3 in the basic variants. The total channel length is equal to $l = 26$.

The energy-supply zones have either a nearly rectangular shape or that of a corner (sweptback) and consist of integral grid cells. The sweepback of the zones is determined by the shift of one layer (stretched along x) of the cells by n cells (sweepback parameter) from the neighboring layer. The energy supply is so fast each time that we disregard the change in the gas density and velocity over the corresponding very short period of time. The density of the gas energy e in the ith zone increases by

$$\Delta e\,(x, y) = \Delta e\,, \quad x_{i-1} < x < x_i\,, \quad y_{i-1} < y < y_i\,, \qquad i = 1, 2, ..., \quad x < x_{i-1} \cup x > x_i \cap y_{i-1} < y < y_i\,,$$

The value of Δe is determined by comparison of the power of the supplied energy and the power in its continuous supply in a quantity corresponding to the complete combustion of hydrogen. The energy power supplied to a unit volume is taken to be

$$q = \rho u\,\frac{Q}{\Delta x}\,,$$

where $\Delta x = x_i - x_{i-1}$. This power is supplied uniformly along x in the prescribed range $[x_{i-1}, x_i]$.

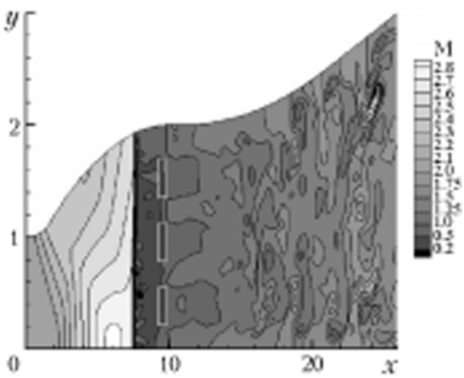
In the pulse periodic regime of energy supply, in accordance with what has been stated above, the energy density supplied in one pulse is equal to

$$\Delta e = \rho u\,\frac{Q}{\Delta x}\,\Delta t\,.$$

To solve these equations we prescribed the parameters of unperturbed flow at the channel inlet (for $x = 0$). For supersonic velocities, extrapolation is used at the outlet ($x = l$). On the upper channel wall (for $y = y(x)$), we set the nonflow condition: the normal velocity component is $v_{\text{n}} = 0$. In the plane of symmetry (for $y = 0$), we specify the symmetry conditions: the problem is solved for the upper half of the channel. When the energy supply is absent, we use the parameters of stationary gas flow as the initial conditions.

The finite-volume scheme decreasing the total variation (TVD reconstruction) is used at the intervals between the instants of energy supply to find the numerical solution. The fluxes at the cell boundaries are computed by the method of [10]. Time integration is carried out by the Runge–Kutta method of the third order. The computational grid in the physical domain is geometrically adapted to the channel contour; it is rectangular in the canonical domains; the number of computational nodes is 400×280 in the basic calculations. The stationary solution with no energy supply, which is used as initial conditions, is found by the same method with a relative error in the mass flow rate of 10^{-6}.

Calculation Results. The basic results have been obtained for the following variant. A uniform flow with Mach number M = 2, a gas density $\rho = \gamma = 1.33$, and a pressure $p = 1$ is prescribed at the channel inlet. The quantity Q determining the power input is taken to be $Q = 9.85$. These values of the parameters correspond to the flight conditions with Mach number $\text{M}_\infty = 0$ at an altitude where the air temperature is $T_\infty = 218$ K in a tenfold compression



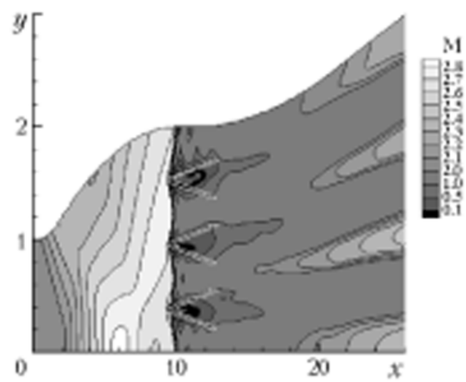


Fig. 1. Distribution of Mach numbers in the channel in energy supply in three rectangular zones after 10,000 periods.

Fig. 2. Distribution of Mach numbers in periodic flow in energy supply in three zones having the shape of a thin corner.

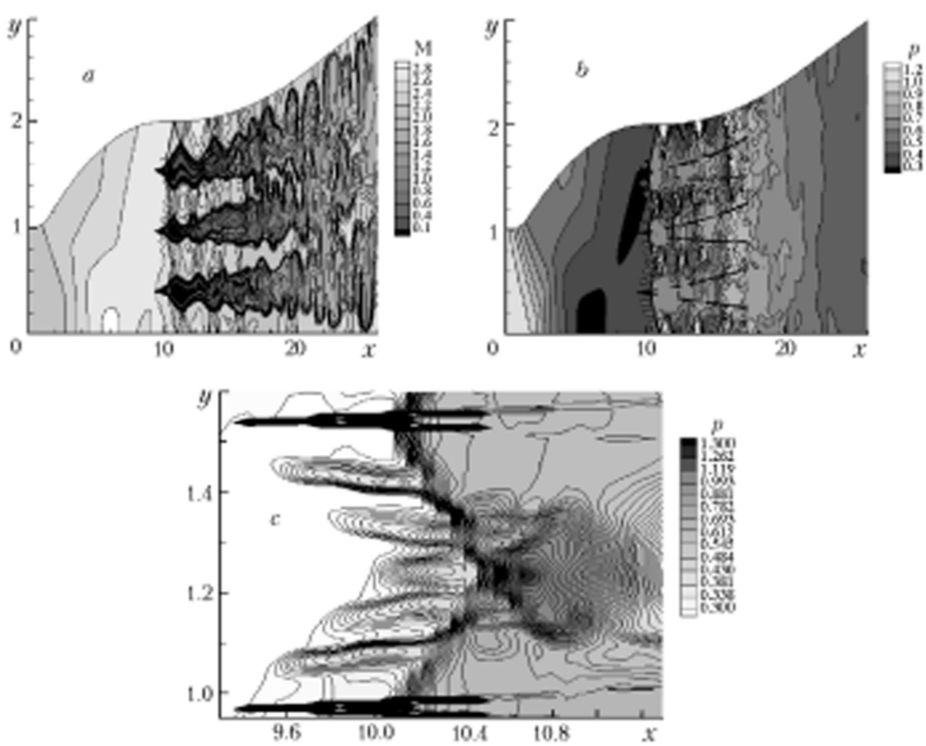


Fig. 3. Distribution of the parameters of flow in energy supply in three zones with a sweepback parameter n = 5: a) distribution of the Mach number; b) pressure field; c) pressure field in the separated subregion.

of the jet cross section in the air intake. The calculation results given below have been obtained for the period of energy supply Δt = 0.1. The sweepback parameter n ranged within n = 0–10.

A "stationary" normal shock localized closer to the inlet is formed in energy supply in the narrower part of the channel in several rectangular zones (n = 0) not overlapping the channel cross section and in the zones repeating the obtuse corners at the beginning of the section of constant cross section (in the "combustion chamber") [5, 6]. This result has been obtained for the case where the zones overlap the entire channel or half the channel (by analogy with the combustion of hydrogen, this corresponds to combustion with an excess-air coefficient α = 1 or 2). Such a character of flow is presented in Fig. 1 (n = 0 and α = 2; the boundaries of the zones of energy supply are shown as a

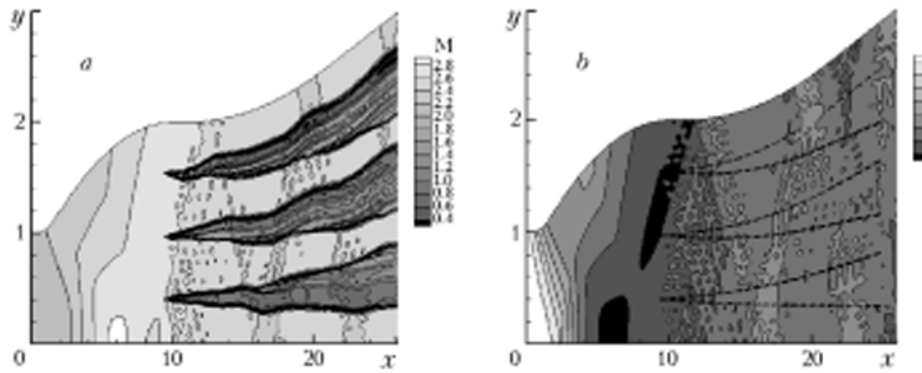


Fig. 4. Distribution of the parameters of flow in energy supply in three zones with a sweepback parameter n = 10: a) distribution of the Mach number; b) pressure field.

white line). The upstream movement of the compression wave and the formation of the stationary normal shock observed in Fig. 1 correspond to the experimental dependence of the pressure on time; this dependence was obtained in the model duct of a hypersonic ramjet engine in the combustion of hydrogen [11]. Contact discontinuities which separate the flows traversing the zones of energy supply and bypassing them are unstable. A periodic regime of flow is not established. An analogous character of flow is obtained for the case where the zones of energy supply have the shape of acute corners and overlap the entire cross section of the channel.

Periodic smooth flow with an "attached" shock can occur in energy supply in the zones in the shape of acute corners (see Fig. 2).

Other regimes of flow can occur with increase in the sweepback of the energy-supply zones. Figure 3 gives the distributions of the Mach number (a) and the static pressure (b) for the sweepback parameter n = 5 (the form of the energy-supply zones can be judged from the black dashed lines in Fig. 3b, i.e., from the tracks of energy supply, α = 2). The occurring (weaker) shock waves do not go upstream: they cross the zones of energy supply and have a curvilinear shape. Unlike the previous variant, these shock waves are nonstationary: we observe a slight irregular variation of their position. The reason for such behavior is that shock waves on the portions behind the "tops" of the energy-supply zones are normal and flow behind them is subsonic. The perturbations of the flow, brought about by the irregular oscillatory motion of the contact discontinuities which separate the flows that transversed the energy-supply zones and bypassed them, propagate upstream, displacing the position of the shock waves. Furthermore, the nonstationary position of the shock waves is influenced by the transverse compression waves propagating ahead of the shock waves (Fig. 3c). Reciprocating vortex flow immediately behind the shock waves is observed in subsonic flow regions. The interaction of the shock waves reflected from the channel wall, the plane of symmetry, and each other with the contact discontinuities leads to a sharp kink of the latter (see Fig. 3a). Subsequently, just as in the first variant, the oscillation motion of the contact discontinuities develops because of their instability. It is noteworthy that the subregions of subsonic velocities occupy a small part of the flow region, which is due to both the sweptback shape of the zones of energy supply and the acceleration of the flow as a result of the energy supply and the divergence of the channel.

A flow regime in which no shock waves appear can occur, as the sweepback of the energy-supply zones increases further. Figure 4 gives the distributions of the Mach number and the static pressure for the sweepback parameter n = 10 (black dashed lines in Fig. 4b are the tracks of energy supply). Periodic flow is established in this variant. In Fig. 4b, it is seen that the shock waves are absent (the parametric distributions are given just before the next supply of energy). Subregions of sub- and supersonic velocities are observed in the region between the contact surfaces connected with one zone of energy supply (see Fig. 4a). The entire flow remains virtually supersonic. An important factor of the occurrence of such a flow regime is the extension of the energy-supply zone to the divergent part of the channel. In any case the transient regime of flow with weak nonstationary waves occurs in six, not in three, zones (n = 5) located in a part of the channel of constant cross section (in the "combustion chamber") upon the supply of the same quantity of energy (α = 2) as that in the previous variants.

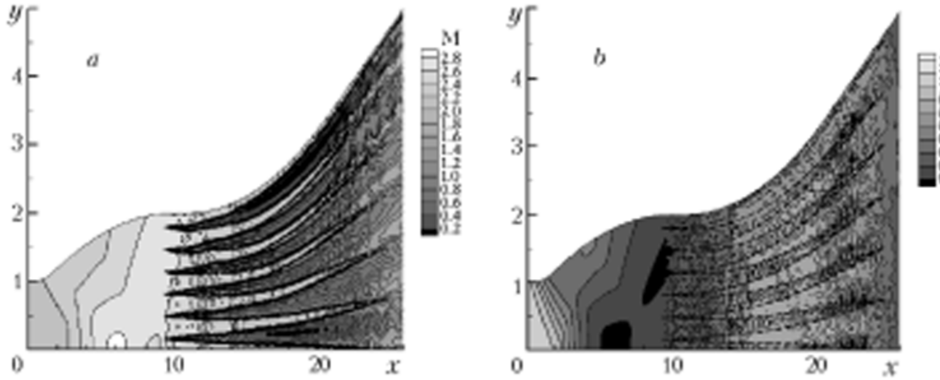


Fig. 5. Distribution of the parameters of flow in energy supply in six zones with a sweepback parameter $n = 10$: a) distribution of the Mach number; b) pressure field.

The transient substantially nonstationary regime of flow occurs for the energy supply in six zones virtually overlapping the channel ($\alpha \approx 1$) and the sweepback parameter $n = 10$. A shock-free regime has been obtained only with increase in the outlet cross section to 5. The results of calculation of this regime are presented in Fig. 5. Flow is aperiodic because of the instability of the contact surfaces. In most of the channel region (with energy supply; $x \approx 13$–24), the pressure changes only slightly, remaining within $p \approx 0.7$–0.8 (see Fig. 5b). The gas flow in individual jets is alternatively accelerated and decelerated (with a change of one-tenth in the local Mach number in supersonic jets; see Fig. 5a), which is due to the transverse vibrations of contact discontinuities.

Conclusions. It has been established that we have a considerable rearrangement of supersonic flow in a channel modeling a hypersonic jet engine in the case of the supply of energy equal to that released in the complete combustion of hydrogen in air. There are different regimes of flow. The characteristic features of the flow structure in the first regime of flow (sweepback parameter $n = 0$) are the normal shock in the narrower forepart of the channel, the subsonic region behind it in which "combustion" occurs, the portion of acceleration of the flow, and transition to supersonic velocities. The instability of contact discontinuities separating the gas flows, to which energy is supplied and is not supplied, is observed. In the second limiting case, for energy supply in the zones having the shape of a thin corner with a large sweepback ($n = 10$) and extending to the divergent part of the channel and an excess-air coefficient of the order of two, we observe a periodic regime of a universally supersonic flow with the absence of shocks. There is the transient regime of flow with relatively weak curvilinear shocks which cross the energy-supply zones. The position of these shocks can vary with time. The formation of a periodic regime with "attached" shocks is possible. When $\alpha = 1$ the shock-free regime remains aperiodic.

NOTATION

a, dimensionless velocity of sound referred to a_0; a_0, velocity of sound in the incident flow; d, inlet half-width of the channel; e, total energy of a unit volume of the gas, referred to $\rho_0 a_0^2$; **F**, vector function of the flow along the x axis; **G**, vector function of the flow along the y axis; $g(t) = \sum_m \delta(t - m\Delta t)$; i, No. of energy-supply zone (or portion of the zone in the shape of a corner); l, total channel length referred to d; M, Mach number; M_∞, flight Mach number; m, number of periods of energy supply; n, sweepback parameter; p, pressure referred to $\rho_0 a_0^2$; p_0, dimensional pressure at the channel inlet; **Q**, vector source function; Q, quantity of heat released in stoichiometric combustion of hydrogen in a unit mass of air and referred to a_0^2; q, power input to a unit volume of the gas, referred to $\rho_0 a_0^3/d$; T_∞, temperature of air at the flight altitude, K; t, time referred to d/a_0; **U**, vector function of the sought quantities; u and v, components of the vector of gas velocity along the x and y axes respectively and referred to a_0; x and y, Cartesian coordinates along the channel and across it and referred to d; α, excess-air coefficient; γ, adiabatic exponent; Δe, energy supplied to a unit volume of the gas; Δt, period of energy supply; Δx, length of the ith zone of energy supply; $\Delta(t)$, impulse Dirac function; ρ, dimensionless gas density in the flow, referred to ρ_0; ρ_0 is determined from the condition $p_0 = \rho_0 a_0^2$. Subscripts: 0, dimensional parameters at the channel inlet; ∞, parameters at the flight altitude; n, normal.

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